Applying the Linear Matrix Inequality for Hybrid Fuzzy/H-infinity Control of Active Structural Damping

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ABSTRACT

This paper proposes a design method of H^{∞} control performance for structural systems using the Tagagi-Sugeno (T-S) fuzzy model. The structural system with tuned mass damper is modeled by T-S type fuzzy model. Through using the parallel distributed compensation (PDC) scheme, we design a nonlinear fuzzy controller for the tuned mass damper system. According to the control system, a sufficient stability condition is derived in terms of Lyapunov theory and this control problem is reformulated into solving the linear matrix inequalities (LMI) problem. Furthermore, the tuned mass damper is designed according to the first mode of frequency of the control system and then the fuzzy controller is found via Matlab LMI toolbox to stabilize the structural system. A simulation example is given to show the feasibility of the proposed fuzzy controller design method.

Key words: H^{∞} control, Linear matrix inequality, Fuzzy control

應用線性矩陣不等式於主動結構阻尼之混合模糊H-infinite控制研究

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摘 要

本研究提出T-S模糊系統與H^{∞}結合之控制器對結構控制之控制設計演算法。以T-S模糊系統模擬，經平行分散式補償(Parallel Distributed Compensation, PDC)流程設計非線性模糊控制器，控制結構裝置調和質塊阻尼器(Tuned Mass Damper, TMD)之振動反應。根據Lyapunov 理論推導穩定控制回饋，並將控制器的設計問題轉化為在數值求解更簡便之求解線性矩陣不等式(linear matrix inequalities ,LMI)問題。最後本研究將以數值模擬實例，以驗證本控制演匴法之可行性。

關鍵字： H^{∞} 控制, 線性矩陣不等式,模糊控制

1. Introduction

In civil engineering, disturbances such as earthquake tremors, wind loadings, and system parameters etc. are always unknown. However, in the seismic design field of structural systems, researchers always considers past representative earthquake records as an input of controlled systems to demonstrate the effectiveness of the proposed control methods (for example, see Aldemir and Bakioglu, 2001; Ghaboussi and Joghataie, 1995; Nagashima et al., 2001; Rao and Datta, 2005; Yang et al., 1995a; Yang et al., 1995b and the references therein). These representative disturbances included El Centro, Tabas, and Taft earthquake etc. The peak ground acceleration recorded by wave profiles almost exceeds the design standard that structures can bear. Some of the above papers placed emphasis on a predictive active control for structural systems. Most of them proposed and demonstrated the effectiveness of their controller design in terms of reducing the response of structures with the representative disturbances. According to the previous reports, if the proposed controller can readily reduce the structure response subjected to a strong earthquake via the demonstration with numerical simulations, the control methods were regarded as quite robust and remarkable.

Fuzzy control recently has increasingly attracted attention because it has been applied successfully to various nonlinear applications. Among them, the famous T-S fuzzy model was proposed by Takagi and Sugeno (1985) to describe nonlinear systems. In this type of fuzzy model, local dynamics in different state space regions are represented by a set of linear sub-models. The overall model of the system is then a fuzzy "blending" of these linear sub-models (Tan *et al.*, 1997). Based on the T-S model, the parallel distributed compensation (PDC) concept was used to design the fuzzy controller of nonlinear systems (Wang *et al.*, 1995). In the PDC concept, the fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts and each control rule is distributively designed for the corresponding rule of the fuzzy model. The overall nonlinear control system is not always stable even if all its subsystems are stable. Some significant stability analysis results have provided design methodologies that can ensure the overall system stability (Tanaka and Sugeno, 1992; Tanaka and Sano, 1994; Tanaka *et al.*, 1996; Wang *et al.*, 1996; Hsiao *et al.*, 2005a; Hsiao *et al.*, 2005b; Chen *et al.*, 2006; Chen, 2006).

Recently, with the increasing research activities in the field of structural control, many control methods have been proposed. Among these methods are the optimal control, pole placement, and sliding mode control etc. Moreover, these control methods have successfully accomplished real applications and structural systems [for example, see Lu *et al.*, 2003; Yeh *et al.*, 1996 and the references therein]. However, as far as we know, the analysis of the stability and stabilization problem of structural systems remains an open area. In the present study, the Chi Chi earthquake with over 1-g gravitational acceleration is used as an input excitation to demonstrate the effectiveness of the controller design. Therefore, this paper may be viewed as a generalization of the controller design for reducing the response under representative disturbances and highlights a newly stability condition for structural systems. For this reason, we proposed here a fuzzy control technique as well as T-S fuzzy model to deal with structural control problem.

The organization of this paper is presented as follows: First, in order to model the structural systems, T-S fuzzy modeling is briefly presented and the equation of the motion of structural systems with tuned mass damper is constructed. Then, the H^{∞} stability criterion is provided for the existence of the T-S fuzzy controllers which achieve the control performance of control system via the Lyapunov theory. In this section, the control problem can be reformulated into a problem of solving linear matrix inequality (LMI). Finally, simulation results show the utility of the proposed fuzzy control methodology, and conclusions are drawn.

2. Recalled Model and Control Methods

As fuzzy logic develops, some mathematical models using fuzzy theories are elaborated on to achieve greater accuracy, dimensionality and also the desire to simplify the structure. Compared with conventional mathematical models, the main advantage of the fuzzy model is the possibility of elaborating them on the basis of a far lesser amount of information concerning a real system and the information can be of an uncertain, fuzzy or inexact character. These fuzzy models include Mamdani, relational, T-S types etc. The most often used type among them is the T-S model for describing controllers. T-S models were described in (Takagi and Sugeno, 1985) for the first time. Tanaka and Wang (2001) proved that any smooth nonlinear control system can be approximated by the T-S fuzzy model with linear rule consequence by a set of flat linear segments. Therefore, T-S fuzzy models were also termed quasi-linear models or fuzzy linear models. Because of this main feature, any smooth nonlinear stated feedback controller can be approximated by the parallel distributed compensation (PDC) controller and the overall fuzzy system can be achieved by fuzzy “blending” of these linear functions through nonlinear fuzzy membership functions. Furthermore, the representation of a plant and controller via T-S type fuzzy model makes it easier to prove the stability of control systems due to the formulation of a locally linear structure. Therefore, T-S fuzzy modeling is employed in this paper to simplify the controller design problem and stability conditions are derived in terms of Lyapunov direct methods in combination with LMI. The controller design procedure is shown in Fig. 1.

First of all, the T-S fuzzy model can have the form of a set of rules as well as membership functions represented below.

R1: IF (z is F_1) THEN ($r = f_1(z)$),

R2: IF (z is F_2) THEN ($r = f_2(z)$),

$\vdots$

Rm: IF (z is F_m) THEN ($r = f_m(z)$),

Then the output can be obtained on the basis of the grade of activation of the particular conclusions f_i, $i = 1, 2, \cdots, m$, which is determined by following formula.

$$r = \frac{\sum_{i=1}^{m} \mu F_i(z) f_i(z)}{\sum_{i=1}^{m} \mu F_i(z)}$$

Where F_i is the fuzzy set; the function $f_i(z)$ can be nonlinear. Then, a nonlinear system can be approximated by this T-S fuzzy model technique. The T-S model consists of a set of If-Then rules. Each rule represents the local linear input-output relation of the nonlinear system and has the following form:

A. T-S Fuzzy Model

Plant Rule *i*:

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_g(t)$ is M_{ig}

THEN $\dot{X}(t) = A_i X(t) + B_i U(t) + E_i \phi(t)$, $i = 1, 2, \cdots, r$ (1)

where M_{ip} ($p = 1, 2, \cdots, g$) is the fuzzy set; $X(t) \in R^n$ is the state vector; $U(t) \in R^m$ is the input vector; r is the rule number; $z_1(t) \sim z_g(t)$ are the premise variables; $A_i \in R^{n \times n}$, $B_i \in R^{n \times m}$ (Takagi and Sugeno, 1985).

B. PDC Design

The fuzzy controller rules have the same premise parts as those of the T-S model. The linear control rule *i* is derived based on the state equation (1) in the consequent part of the *i*th model rule.

Control Rule *i*:

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_g(t)$ is M_{ig}

THEN $U(t) = -F_i X(t)$, $i = 1, 2, \cdots, r$ (2)

where F_i is the local feedback gain matrix. The final control U is inferred using the Sum-Product reasoning method:

$$U(t) = -\frac{-\sum_{i=1}^{r} w_i(t) F_i X(t)}{\sum_{i=1}^{r} w_i(t)} \quad (3)$$

where w_i is the activation degree of the *i*th rule, calculated as: $w_i(t) = \prod_{p=1}^{g} M_{ip}(z_p)$.(Takagi and Sugeno, 1992)

3. Motion Equations of Structural Systems

Assume that the equation of motion for a shear-type-building modeled by an n-degrees-of-freedom system controlled by actuators and subjected to external force $\phi(t)$ can be characterized by the following differential equation:

$$M\ddot{\overline{X}}(t) + C\dot{\overline{X}}(t) + K\overline{X}(t) = BU(t) - M\overline{r}\phi(t) \tag{4}$$

where $\overline{X}(t) = [\overline{x}_1(t), \overline{x}_2(t) \cdots \overline{x}_n(t)] \in R^n$ is an n-vector; $\ddot{\overline{X}}(t)$, $\dot{\overline{X}}(t)$, $\overline{X}(t)$ are acceleration, velocity, and displacement vectors; matrices M, C, and K are $(n \times n)$ mass, damping, and stiffness matrices, respectively; $\overline{r}$ is an n-vector denoting the influence of the external force; $\overline{B}$ is a $(n \times m)$ matrix denoting the locations of m control forces; $\phi(t)$ is the excitation with a upper bound $\phi_{up}(t) \geq \|\phi(t)\|$; U(t) corresponds to the actuator forces (generated via active tendon system or an active mass damper, for example); this is only a static model.

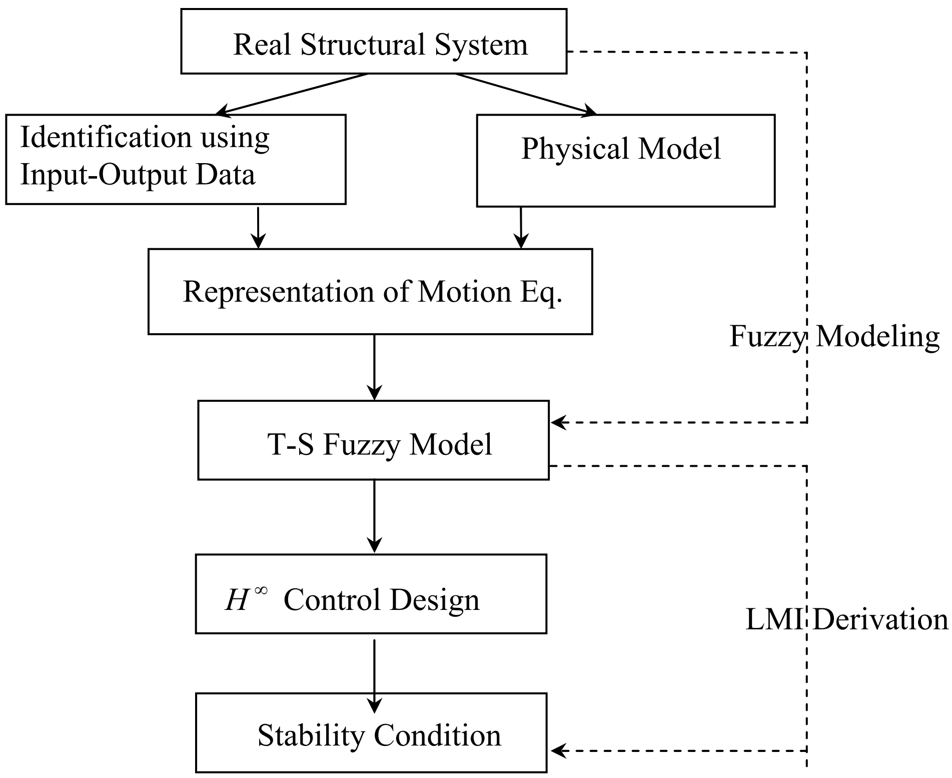

Fig. 1 The complete design procedure

For controller design, the standard first-order state equation corresponding to Eq.(4) is obtained by

$$\dot{X}(t) = AX(t) + BU(t) + E\phi(t) \tag{5}$$

where

$$X(t)=\begin{bmatrix}\bar{x}(t)\\ \dot{\bar{x}}(t)\end{bmatrix}_{2n\times 1},\ A=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}_{2n\times 2n},\ B=\begin{bmatrix}0\\ M^{-1}\bar{B}\end{bmatrix}_{2n\times n},\ E=\begin{bmatrix}0\\ -\bar{r}\end{bmatrix}_{2n\times 1},$$

in which,

$$A_{11}=0,\ A_{12}=I_{n\times n},\ A_{21}=\begin{bmatrix}-\frac{k_1+k_2}{M_1} & -\frac{k_2}{M_1} & & & 0\\ -\frac{k_2}{M_2} & \frac{k_2+k_3}{M_2} & -\frac{k_3}{M_2} & & \\ & -\frac{k_3}{M_3} & \ddots & \ddots & \\ & & -\frac{k_{n-1}}{M_{n-1}} & \frac{k_{n-1}+k_n}{M_{n-1}} & -\frac{k_n}{M_{n-1}}\\ 0 & & & -\frac{k_n}{M_n} & \frac{k_n}{M_n}\end{bmatrix}_{n\times n},$$

$$A_{22}=\begin{bmatrix}-\frac{c_1+c_2}{M_1} & -\frac{c_2}{M_1} & & & 0\\ -\frac{c_2}{M_2} & \frac{c_2+c_3}{M_2} & -\frac{c_3}{M_2} & & \\ & -\frac{c_3}{M_3} & \ddots & \ddots & \\ & & -\frac{c_{n-1}}{M_{n-1}} & \frac{c_{n-1}+c_n}{M_{n-1}} & -\frac{c_n}{M_{n-1}}\\ 0 & & & -\frac{c_n}{M_n} & \frac{c_n}{M_n}\end{bmatrix}_{n\times n}. \tag{6}$$

4. Fuzzy Modeling of Structural System

To discuss the stability of the structural system in advance, Takagi-Sugeno (T-S) fuzzy models and some stability analysis are utilized to approximate the structural system.

The *i*th rule of the T-S fuzzy model, representing the structural system (5) is the following:

Rule i : IF $x_1(t)$ is M_{i1} and $\cdots$ and $x_p(t)$ is M_{ip}

THEN $\dot{X}(t)=A_iX(t)+B_iU(t)+E_i\phi(t)$ (7)

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifier, the dynamic fuzzy model (7) can be expressed as following:

$$\dot{X}(t)=\frac{\sum_{i=1}^{r}w_i(t)[A_iX(t)+B_iU(t)+E_i\phi(t)]}{\sum_{i=1}^{r}w_i(t)} \tag{8}$$

where w_i is the activation degree of the ith rule, calculated as

$$w_i(t) = \prod_{p=1}^{g} M_{ip}(z_p(t)),$$

and
$$h_i(t) = \frac{w_i(t)}{\sum_{i=1}^{r} w_i(t)}$$

Based on Eq. (8) and the PDC described in Eq. (3), the closed-loop control system is (Takagi and Sugeno, 1992)

$$\dot{X}(t) = \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t)h_l(t)[(A_i - B_iK_l)X(t)] + E_i\phi(t) \tag{9}$$

5. H^∞ Control Design via Fuzzy Control

In order to attenuate the influence of the excitation $\phi(t)$ on the state variable $X(t)$ (Chen *et al.*, 1999; Lin and Byrnes, 1996), this section proposes the H^∞ control performance. Hence, in this work, not only the stability of fuzzy control systems is advised but also the H^∞ control performance is satisfied as follows:

$$\int_0^{t_f} X(t)^T QX(t)dt \le X(0)^T PX(0) + \eta^2 \int_0^{t_f} \phi(t)^T \phi(t)dt \tag{10}$$

where t_f denotes the terminal time of the control, P are some positive definite matrices, η is a prescribed value which denotes the effect of $\phi(t)$ on $X(t)$, and Q is a positive definite weighting matrix.

Definition 1(Lu *et al.*, 1998): **LMI Formulation of the Design Specifications**

The linear matrix inequality (LMI) is any constraint of the form

$$F(v) = F_0 + \sum_{i=1}^{m} v_i F_i > 0 \tag{11}$$

where $v = [v_1, v_2, \cdots, v_m] \in R^m$ is the variable vector, and the symmetric matrices $F_i = F_i^T \in R^{n\times n}, i = 0,\cdots,m,$ are given. It can be shown that the solution set $\{v \mid F(v) > 0\}$ may be empty, but it is always convex. Thus, although (11) has no analytic solution in general, it can be solved numerically by efficient numerical algorithms. Many control problems can be reformulated into LMI's and solved efficiently by recently developed interior-point methods (Boyd *et al.*, 1994).

Prior to examination of a typical stability condition for a fuzzy system (9), a stability concept is given below.

Definition 2 (Chen *et al.*, 1999; Khalil, 1992): The equilibrium state $x = 0$ is said to be stable if, for and $R > 0$, there exists $r > 0$, such that if $\|x(0)\| < r$, then $\|x(t)\| < R$ for all $t \geq 0$. Otherwise, the equilibrium pint is unstable.

Essentially, stability, also called Lyapunov stability, which means that the system trajectory can be kept arbitrarily close to the origin by starting sufficiently close to it. Consider a system with disturbances, there is a definition for stability derivation below.

Definition 3 (Khalil, 1992)**:** The solution of a dynamic system are said to be uniformly ultimately bounded (UUB) if there exist positive constants β and κ, and for every $\delta \in (0, \kappa)$ there is a positive constant $T = T(\delta)$, such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| \leq \beta, \forall t \geq t_0 + T . \tag{12}$$

According to the stability concepts above, a stability condition is derived below to guarantee the stability and control performance of the closed-loop control system (9).

Theorem 1: The equilibrium point of fuzzy control system (9) is stable in the large if there exist a common positive definite matrix *P* such that the following inequality is satisfied:

$$(A_i - B_i K_l)^T P + P(A_i - B_i K_l) + \frac{1}{\eta^2} P E_i E_i^T P + Q < 0 \tag{13}$$

with $P = P^T > 0$, for $i < l \leq r$ and $i = 1, 2, \cdots, r$

Proof: Using the Lyapunov function candidate for the fuzzy system (9)

$$V = X^T(t) P X(t) . \tag{A1}$$

The time derivative of *V* is

$$\dot{V} = \dot{X}^T(t) P X(t) + X^T(t) P \dot{X}(t)$$

$$= \{\sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t)\, h_l(t)[(A_i - B_i K_l) X(t)] + E_i \phi(t)\}^T P X(t)$$

$$+ X^T(t) P \,\{\sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t)\, h_l(t)[(A_i - B_i K_l) X(t)] + E_i \phi(t)\} \tag{A2}$$

$$= \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t)\, h_l(t) X^T(t)[(A_i - B_i K_l)^T P + P(A_i - B_i K_l)] X(t)$$

$$+ \phi^T(t) E_i^T P X(t) + X^T(t) P E_i \phi(t) - [\eta^2 \phi^T(t)\phi(t) + \frac{1}{\eta^2} X^T(t) P E_i E_i^T P X(t)]$$

$$+ [\eta^2 \phi^T(t)\phi(t) + \frac{1}{\eta^2} X^T(t) P E_i E_i^T P X(t)] \tag{A3}$$

$$\le \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t)\, h_l(t) X^T(t)[(A_i - B_i K_l)^T P + P(A_i - B_i K_l) + \frac{1}{\eta^2} P E_i E_i^T P] X(t)$$

$$-(\frac{1}{\eta}(PE_i)^T X(t) - \eta\phi(t))^T (\frac{1}{\eta}(PE_i)^T X(t) - \eta\phi(t)) + \eta^2 \left\|\phi_{up}(t)\right\|^2 . \tag{A4}$$

Based on Theorem 1 and (A4),

$$\dot{V} \le \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t) h_l(t) X^T(t)\{-Q\} X(t) + \eta^2 \left\|\phi_{up}(t)\right\|^2$$

$$= -X^T(t) Q X(t) + \eta^2 \left\|\phi_{up}(t)\right\|^2 \tag{A5}$$

Integrating (A5) from $t = 0$ *to* $t = t_f$ yields

$$V(t_f) - V(0) \le -\int_0^{t_f} X(t)^T Q X(t) dt + \eta^2 \int_0^{t_f} \phi(t)^T \phi(t) dt . \tag{A6}$$

From (A1), we get

$$\int_0^{t_f} X(t)^T Q X(t) dt \le X^T(0) P X(0) + \eta^2 \int_0^{t_f} \phi(t)^T \phi(t) dt . \tag{A7}$$

Eq. (A7) is Eq. (10). Therefore, the H^{∞} control performance is achieved.

Lemma 1 (Schur Complements): (Boyd *et al.*, 1994; Horn and Johnson, 1991)

$$\text{The LMI} \begin{bmatrix} Q(x) & S(x) \\ S(x) & R(x) \end{bmatrix} > 0 \tag{14}$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x)$ depends on x is equivalent to

$$R(x) > 0,\ Q(x) - S(x) R^{-1}(x) S^T(x) > 0 \tag{15}$$

In other words, the set of nonlinear inequalities (15) can be represented as the LMI (14).

Remark 1: Theorem 1 can be reformulated into the linear matrix inequality (LMI) problem and efficient interior-point algorithms are now available in Matlab toolbox to solve this problem. Therefore, Theorem 1 is transformed to the LMI by the following procedure.

By introducing new variables $H_{il} = A_i - B_i K_l$, $W = P^{-1}$, and $Y_{il} = H_{il} W$. Eq. (12) can be rewritten as follows by Lemma 1:

$$\begin{bmatrix} Y_{il} + Y_{il}^T + \frac{1}{\eta^2} E_i E_i^T & W \\ W & -Q^{-1} \end{bmatrix} < 0 \tag{16}$$

This paper proposes a stability condition for a nonlinear structural system based on both linear matrix inequality (LMI) transformation and the T-S fuzzy model. Although the controller design problem can be transformed into a solvable LMI problem, the control approach has to be enhanced to be effective for real engineering applications. Here, we consider the model-based fuzzy control methods presented in this paper for a four-story building with TMD system.

6. A Simulation Example

The applications of the model-based fuzzy control methods presented in this paper are illustrated in this section. The complete design procedure of the real structural system is described below. First, a four-story building of real structural system is considered. The mass, stiffness of each floor mass is 345600kg, $3.1*10^8$ nt/m, and damping ratio is 0.02. According to the above system parameters, we assume mass and stiffness of Tuned Mass Damper(TMD) are $m_d = 20736$ kg, $k_d = 2240000$ N/m, and we will have a damping ratio of TMD = 0.02, the nature frequency of the 4-story building, $w_s(Hz) \Rightarrow \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{Bmatrix} 1.65 \\ 4.77 \\ 7.31 \\ 8.96 \end{Bmatrix}$ and the nature frequency of the building with TMD, $w_d(Hz) \Rightarrow \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} = \begin{Bmatrix} 1.53 \\ 1.79 \\ 4.78 \\ 7.31 \\ 8.96 \end{Bmatrix}$. The E-W component of the Taiwan Chi Chi earthquake record in 1999 scaled to a maximum ground acceleration of 1g is used as input excitation shown in Fig. 2, and its duration is 35 seconds.(Ellsworth ,2004)

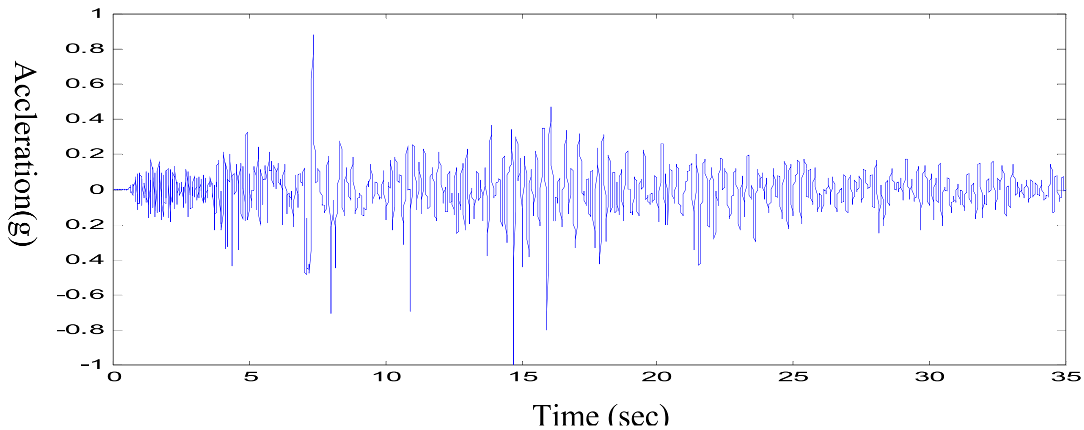

Fig. 2. East-West component of the Chi Chi earthquake

Then, T-S fuzzy models for the structural system are established. To minimize the design effort and complexity, we try to use as few rules as possible. The membership functions for Rules 1 and 2 are $\exp[-x_{12}^2(t)]$ and $1\text{-}\exp[-x_{12}^2(t)]$ which are plotted in Fig. 3. Hence, the structural system is represented by the following fuzzy model:

$$A = 10^3 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.001 & & & & \\ 0 & 0 & 0 & 0 & 0 & & 0.001 & & & \\ 0 & 0 & 0 & 0 & 0 & & & 0.001 & & \\ 0 & 0 & 0 & 0 & 0 & & & & 0.001 & \\ 0 & 0 & 0 & 0 & 0 & & & & & 0.001 \\ -1.794 & 0.897 & 0 & 0 & 0 & -0.002 & 0.0012 & & & \\ 0.897 & -1.794 & 0.897 & 0 & 0 & 0.0012 & -0.002 & 0.0012 & & \\ 0 & 0.897 & -1.794 & 0.897 & 0 & & 0.0012 & -0.002 & 0.0012 & \\ 0 & 0 & 0.897 & -0.904 & 0.0065 & & & 0.0012 & -0.0012 & -0 \\ 0 & 0 & 0 & 0.108 & -0.108 & & & & 0.0004 & -0.0004 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0289 & 0.4823 \end{bmatrix}^T * 10\text{ e-4}.$$

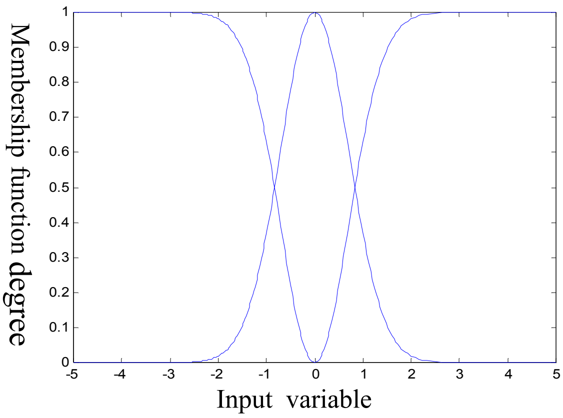

Fig. 3. Membership functions of T-S fuzzy models.

The next step is to design a model-based fuzzy controller via the concept of the PDC scheme. The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. In order to guarantee the H infinity control performance, the optimal values η and Q should be suitably selected in Theorem 1. Based on Remark 1, the common solutions W and Y_{il} are obtained via the Matlab LMI optimization toolbox. Following transformation, there will exist a positive definite matrix $P\,(=W^{-1})$ and feedback gain $K(=Y_{il}W^{-1})$ described below to satisfy the stability condition in Theorem 1.

K =[-1.9115e+014 1.4386e+015 -8.9562e+014 -1.0984e+012 7.8393e+007 8.5543e+014
-4.2719e+014 -1.962e+012 1.7109e+009 2.0611e+008],

P =

$$\begin{bmatrix} 7.609\times10^{8} & -4.452\times10^{8} & 4.256\times10^{7} & -39485 & 37.401 & 1.377\times10^{6} & -1.424\times10^{6} & 89358 & 1510.6 & 91.275 \\ -4.452\times10^{8} & 3.158\times10^{8} & -6.198\times10^{7} & 58560 & -16.331 & 2.618\times10^{7} & -1.271\times10^{7} & -9957 & -645.87 & -39.49 \\ 4.256\times10^{8} & -6.198\times10^{7} & 2.721\times10^{7} & -26364 & -1.689 & -1.797\times10^{7} & 8.98\times10^{6} & 36866 & -76.046 & -4.284 \\ -39485 & 58560 & -26364 & 338.47 & 0.0016 & 18869 & -9222 & -181.2 & 0.0759 & 0.005 \\ 37.401 & -16.331 & -1.689 & 0.0016 & 1.273\times10^{-5} & 1.583 & -0.847 & -0.001 & 0.0002 & 1.34\times10^{-5} \\ 1.377\times10^{6} & 2.618\times10^{7} & -1.797\times10^{7} & 18869 & 1.583 & 1.828\times10^{7} & -9.159\times10^{6} & -19581 & 73.463 & 4.283 \\ -1.424\times10^{6} & -1.271\times10^{7} & 8.98\times10^{6} & -9222 & -0.847 & -9.159\times10^{6} & 4.592\times10^{6} & 9562 & -39.575 & -2.313 \\ 89358 & -9957 & 36866 & -181.2 & -0.001 & -19581 & 9562 & 191.2 & -0.09 & -0.004 \\ 1510.6 & -645.87 & -76.046 & 0.0759 & 0.0002 & 73.463 & -39.575 & -0.085 & 0.016 & 0.001 \\ 91.275 & -39.49 & -4.284 & 0.005 & 1.34\times10^{-5} & 4.283 & -2.313 & -0.004 & 0.001 & 5.85\times10^{-5} \end{bmatrix}.$$

Therefore, based on Theorem 1, the structural system with the TMD equipped under a disturbance is guaranteed to be stabilized by the proposed T-S fuzzy controllers. Therefore, simulation results in Table 1 show the maximum response of the first, second, third, fourth floor with and without input control. The displacement time histories of the 1th, and 4th floor with and without model-based control are presented in Figs. 4-5. From Figs. 4-5, we see that the stability of structural system is ensured because the trajectories from nonzero initial states both approach to zero under an earthquake excitation. The effectiveness and the feasibility of the proposed controller design method are demonstrated in this example.

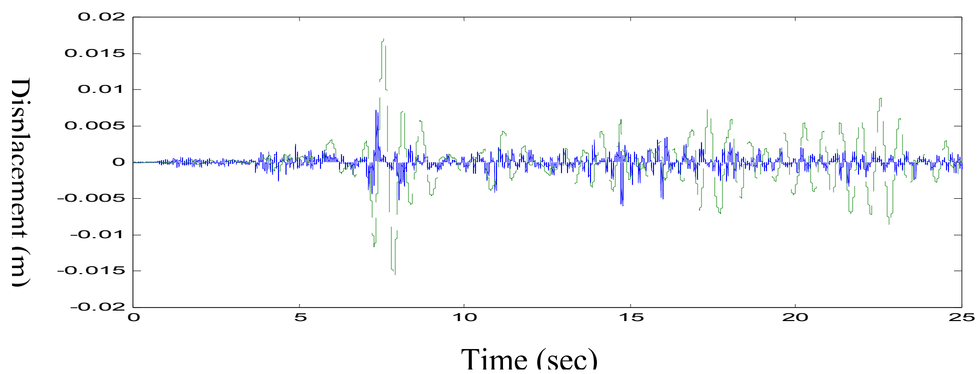

Fig. 5. Time histories of response quantities of the first floor

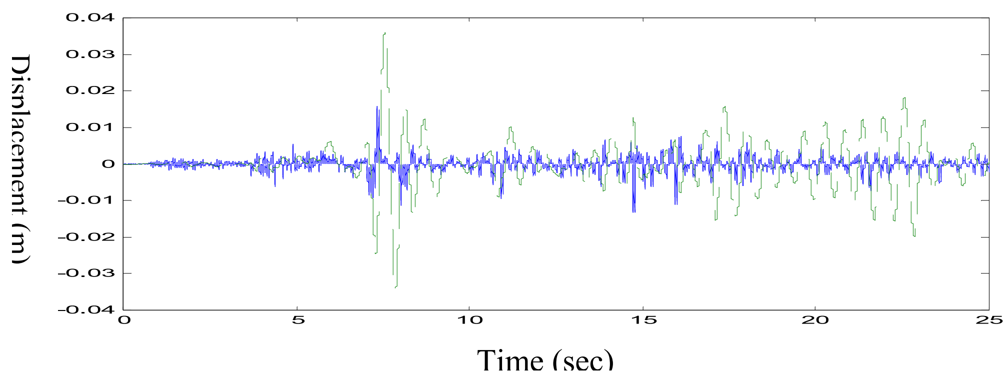

Fig. 6. Time histories of response quantities of the 4th floor.

Table 1. Maximum response with and without input control.

	No control U_{max}=0 kn		Model-based control U_{max}=4.161 kn	
	x_j(m)	$\ddot{x}_j$ (m/s^2)	x_j(m)	$\ddot{x}_j$ (m/s^2)
1f	0.0175	10.75	0.0072	0.542
2f	0.027	13.21	0.0139	0.521
3f	0.0361	10.81	0.0197	0.523
4f	0.0409	14.75	0.0129	0.455

7. Conclusions

The objective of this paper is to develop an efficient fuzzy control algorithm on stability problems of the structural systems presented by T-S type fuzzy model. The fuzzy modeling and the PDC control design are recalled to represent a structural system and reduce the system response under disturbances. In the design procedure, the fuzzy system is represented as a family of local state space models, and we construct a global fuzzy logic controller by blending all such local state feedback controllers. In this study, a stability criterion of H^{∞} control performance is derived for the fuzzy system based on Lyapunov theory. Based on this criterion, the fuzzy controller design problem can be reduced into an LMI problem. The effectiveness and the feasibility of the proposed controller design method is demonstrated through numerical simulations on a four-story shear building under seismic excitation, like Taiwan Chi Chi earthquake which occurred in 1999. This proposed methodology could be applied in practical structural system from the example.

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