智慧型垃圾桶決策模型進化算法於結構最適化設計

Optimum Designof a Planar Frame Bridge Using an Intelligent Can Decision-Making Model Evolution Algorithm

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摘要

基於差分進化演算法及垃圾桶決策模型的框架下，本文利用智慧型垃圾桶決策模型進化演算法（IGCMEA）來模擬人類社會組織的決策過程決策。在決策過程中，當面對如不明確的目標和技術，參與者周轉等，所有參與各方代表將交流、爭論、妥協和相互適應，才能找到解決問題的方案。分組會議將促成選擇一個更客觀、合理及有效的方式的最佳解決方案。
最後，本文使用 IGCMEA 比較差分演算法(DE)與垃圾桶決策模型演算法(GCMEA)，分別對 7 個 100 維標竿(benchmark)函數進行最適化測試，呈現高效和滿意的結果。本文還使用 IGCMEA 對平面構橋的結構進行最適化，結果比文獻使用最速下降法還好，說明 IGCMEA 為強而有效力的方法。

關鍵詞：最適化設計、差分進化算法、垃圾桶決策模型、結構最適化設計。

Abstract

Based on differential evolution algorithms and a framework of garbage can decision-making model, an Intelligent Garbage Can Decision-Making Model Evolution Algorithm (IGCMEA) was used to simulate the decision-making process in human social organizations. In a decision-making process, when faced with issues such as unclear goals and technologies, participators turnover, etc., representatives of all participating parties will communicate, argue, compromise and adapt with each other, in order to find a solution to the problems. Group meetings are conducted to choose the best solution in a more objective, reasonable and efficient way.

At last, by using IGCMEA to compare with the Differential Evolution algorithm (DE) and Garbage Can Decision-Making Model Evolution Algorithm (GCMEA) to carry out an optimization test on the seven benchmarkfunctions, achieving an efficient and satisfactory result. Optimizing the structure of plane frame bridges using IGCMEA and got a better result than using the steepest descent method as in the literature, illustrating the superior power of IGCMEA.

Keywords: optimum design, differential evolution algorithms, garbage can decision-making model, design optimization of structures

1. Introduction

Over the years, the mathematical programming method in which the optimum is searched for in the direction of the gradient of the objective function under constraints has been widely applied in engineering and business management for optimization problems. Particularly, it is studied in the optimal design problems in structural engineering, such as size design, shape design and topology design problems, under the constraints of stress, displacement and other structural responses, as well as the size ratio. However, when faced with large-scale structural engineering design problems where too many variables exist and the constraints are complex, it becomes difficult to analyze the gradient by sensitivity [1, 2].

During the exploration of optimization methods, experts have found that linear programming, nonlinear programming, integer programming, dynamic planning and other traditional algorithms cannot effectively solve the optimization problems in complex large-scale practical engineering problems. Consequently, researchers introduced the human genetic model, animal foraging and migration behavior in natural ecology, and the concept of human social behavior, in order to develop numerical simulations of the evolution algorithms.

In 1975, Holland introduced the concept of natural selection to the algorithms and proposed Genetic Algorithms (GA) [3]. This heuristic algorithm has become a popular research topic during the recent three decades. Later, other bio-inspired population algorithms gradually emerged [4, 5].

Most scholars studying the garbage can decision theory [6] focus on finding an accurate description of the complicated organization and decision-making. They rarely explore the hidden motivations behind the phenomenon. An accurate description of the decision-making process is certainly necessary, but this action is only the starting point for constructing a systematic decision-making theory.

Decision-making is an important part of administration. Theoretical models of administrative decision-making include the following three:

(1)、Classical Model:

Proposed by Frederick Winslow Taylor in 1947 [7], its basic concept approaches an optimistic strategy, assuming that decision-makers are fully rational and use the most promising way to achieve their goals. Their results are perfect. Its main features are:

(a) Identify the problem.

(b) Establish goals and objectives.

(c) Find possible solutions（alternatives）.

(d) Analyze possible consequences.

(e) Evaluate all possible solutions based on the established goals and objectives.

(f) Select the best option.

(g) This decision is practical and assessable.

(2)、Administrative Model:

Proposed by Simon in 1974 [8], its basic concept approaches a satisfactory strategy, assuming that, limited by the existing knowledge, resources and information, it is impossible for decision-makers to make perfect decisions. They can only try to achieve satisfactory results. Its main features are:

(a) Usually goals are set before decisions are made.

(b) Decision-making is a process of goal-means analysis; decision-making is a recursive action; goals may be adjusted during the analysis of consequences.

(c) The key measure of a decision is whether it achieves the goals satisfactorily.

(d) Keep exploring the problem until a reasonable decision（alternative） isdetermined.

(e) Rely on theory and experience.

(3)、Incremental Model:

Proposed by Lindblom in 1959 [9], its basic concept is like a constant comparison strategy under a certain threshold. It assumes that, limited by the existing knowledge, resources and evidence, decision-makers can make neither perfect nor satisfactory decisions. Only through constant comparison can they identify the feasible decision that is most suitable for the current situation. Its main features are:

(a) Goals and alternatives are usually determined simultaneously, so the goal-means analysis is not appropriate.

(b) A good decision means that the decision-makers agree to drop other means for this decision.

(c) Many alternatives and results are dropped after consideration, leaving only one in line with the current situation.

(d) Analysis is also subject to the differences between the existing situation and the plan.

(e) The solution is to avoid theory, constantly choosing the more concrete and practical options.

Lindblom said that when the problem to be solved is of high complexity, uncertainty and full of the controversy, the incremental model is probably the only viable decision model. This decision model is like crossing a river by feeling the stones. We can think of this model as the decision-makers being aware of the goals, but uncertain about the actions required to achieve them. They assess the results after reaching an intermediate stage and then adjust the direction to move on. It is very similar to the real decision making process in a human social organization. The Garbage Can Model is closest to the incremental model. In this spirit, we can infer that the decision-making process in an organization is like an evolving process.

Considering the above, this study attempts to interpret the evolution of the population system in population_based algorithms by the Garbage Can Model. Based on the differential evolution algorithm framework, we propose the Intelligent Garbage Can Model Evolution Algorithm and apply it to practical structural engineering optimization problems.

2. Differential Evolution Algorithm (DE)

The differential evolution algorithm [10] was proposed by Rainer Storn and Kenneth Price in 1997. It uses simple arithmetical calculations, as well as the mutation and crossover operators from GA. Through individuals of different generations competing with each other, the population evolves. The algorithm converges quickly and has strong optimization capability [11, 12]. Individuals from one generation are replaced or evolved to the next generation. The three basic operations for individuals are mutation, crossover and selection. The steps of the differential evolution algorithm are:

(1) Initialization

Create NP individuals:

$$X_i^G = X_{\min} + \rho_i (X_{\max} - X_{\min}), \qquad i = 1,2,\ldots, NP \tag{1}$$

where X_i^G is the vector of individual i of generation G and ρ_i is a random number uniformly distributed between (0, 1).

(2) Evaluation

Evaluate the objective values (fitness) of all individuals

(3) Mutation

Mutation is achieved by a disturbance vector. Essentially, it is obtained by a base vector plus the product of a factor F, known as the mutation factor, and a difference vector, which the difference of two or more random vectors. As shown in Figure 1. , the base vector is X_{r1}^G and the difference vector is the difference between X_{r2}^G and X_{r3}^G. The three vectors are related to randomly selected individuals $r1, r2, r3$.

The mutation vector formula is as follows:

$$V^{G+1} = X_{r1}^{G} + F(X_{r2}^{G} - X_{r3}^{G}) \quad (2)$$

where F is the mutation factor between (0, 2) and $r1, r2, r3$ are different individuals, randomly selected from the generation G. It should be noted that X_{best} is the best individual in the generation G.

DE mutation strategies [10]:

a.DE/best/1 $X_{best} + F(X_{r1} - X_{r2})$ (3)

b. DE/rand/1 $X_{r1} + F(X_{r2} - X_{r3})$ (4)

c. DE/rand-to-best/1 $X_{r1} + F(X_{best} - X_{r1}) + F(X_{r2} - X_{r3})$ (5)

d. DE/best/2 $X_{best} + F(X_{r1} + X_{r2} - X_{r3} - X_{r4})$ (6)

e. DE/rand/2 $X_{r5} + F(X_{r1} + X_{r2} - X_{r3} - X_{r4})$ (7)

The user should choose the strategy that best fits to problem from the above strategies to generate mutation vectors.

The individual vector of generation G is as follows:

$$X_{i}^{G} = (X_{1i}^{G}, X_{2i}^{G}, \ldots, X_{ni}^{G}) \quad (8)$$

The mutation vector is as follows:

$$V^{G+1} = (V_{1}^{G+1}, V_{2}^{G+1}, \ldots, V_{n}^{G+1}) \quad (9)$$

(4) Crossover

Crossover is also conducted at random.

The child vector generated by crossover between X_{i}^{G} from parent generation and V^{G+1} is:

$$y = (y_1, y_2, \ldots, y_n), \quad y_j = \begin{cases} V_{j}^{G+1}, if & r_j \le CR \quad or \quad j = l \\ X_{ji}^{G}, if & r_j > CR \quad and \quad j \ne l \end{cases} \quad (10)$$

where $j = 1,2,3,\ldots,n$, random integer $l \in \{1,2,\ldots,n\}$, random number $r_j \in U(0,1)$, and crossover constant (CR) satisfies $0 \le CR \le 1$.

(5) Evaluation and Selection

Individuals from a parent generation and child generation after mutation and crossover compete with each other. The best ones are retained.

$$(X_i^{G+1}) = \arg\min\{F(X_i^G), F(y)\} \quad (11)$$

To find the best individual of next generation:

$$(X_{best}^{G+1}) = \arg\min\{F(X_i^G)\} \quad (12)$$

where $F(X)$ is the value of the objective function (fitness value).

(6) Repeat steps (3) to (5) until the objective function value (fitness value) reaches the expectation, or the number of generations reaches the maximum.

3. Intelligent Garbage Can Decision-Making Model Evolution Algorithm (IGCMEA)

3.1 Garbage Can Decision-Making Model

Based on Simon's bounded rationality [13], March and Cohen argued that decision-makers, who are limited by the inadequate external information and human subjective feelings that affect rational judgments, can never make the perfect decision. In 1972 [6], they proposed that the organization of an actual decision-making process is a different decision model: the Organized Anarchies Model. This is a non-rational model that includes three main characteristics:

(1) Problematic Preferences

Decision-makers have inconsistent preferences for the problems and goals and their preferences can only be discovered through actions. Therefore, these preferences cannot be the basis for actions.

(2) Unclear Technology

Organizers only know that something needs to be improved during decision-making, but they don’t know what individuals should do to improve. Therefore, they have to utilize trial and error methods, using their personal knowledge.

(3) Fluid Participation

If the issues are controversial and a long time investigation is required before making any decision, it is quite probable that the final decision-makers are not the same group of people as at the beginning. Decision-makers can also come from various perspectives, topics of interest and all walks of life.

In such ambiguous situations, each decision-making process is regarded as a receptacle or garbage can, in which decision-makers, issues, and solutions are represented by garbage. This non-rational decision-making model is called the Garbage Can Model (GCM). In this model, decisions are often determined by four forces, known as the governing variables of the framework of Garbage Can Model: problems/objectives, solutions, participants, and opportunities.

According to the above three ambiguities in the decision-making process, we further propose solutions to these three situations and clarify the issues under the guidance of our goals:

- Problematic preferences: we must overcome controversies with an integrated object model.
- Unclear technology: we must think outside the box and embrace creativity.
- Fluid participation: we must increase the exchange of ideas and grasp any opportunity to solve the problem.

Combining the idea of the group meeting in the decision-making process with the evolution algorithm of Garbage Can Model developed in this paper, we propose the Intelligent GCM Evolution Algorithm (IGCMEA).

3.2 IGCMEA

Intelligent Garbage Can Model Evolution Algorithm is based on differential evolution algorithm, taking the Garbage Can Model as the evolution logic. We make mapsbetween the difference evolution algorithm and the garbage can modelas follows:

individuals in a population → participants in an organism;
search space → solution (alternatives) space;
mutation strategy → garbage decision model;
objective (fitness) function → satisfaction function;

X_{best} → the most satisfied evolutionary solution (alternatives).

After evolution of several generations, the best individual is the most satisfactory decision of the participants in the organization.

For the mutation operations, IGCMEA uses a diverse selection model. It places several mutation strategies in the can, named 'mutation-strategy can'. The mutation strategy of each generation is then randomly selected from the can. Furthermore, when the population matures, the grouping search strategy is used, namely splitting the parent population into several child populations and integrating the search results of each child population. In other words, when the population matures, it identifies the promising space of each variable of each child population then searches separately in each promising space. For example, if the population contains 800 individuals, 400 of which are good, the promising space is defined by the intervals of each variable of these 400 individuals. Therefore, in this study after every few generations, known as the evaluated generation number, the parent population is divided into three child populations and each child population is confined by a search space, as shown in Figure 2.:

(1) The first child population: the search space U^1 is the initial search space,

$U_i^0 = [\underline{x}_i^0, \overline{x}_i^0]$, $i = 1,2,...,n$. $\underline{x}_i^0, \overline{x}_i^0$ are the lower and upper bounds, respectively.

(2) The second child population: the search space U^2 is the promising space

$U_i^p = [\underline{x}_i^p, \overline{x}_i^p]$, which is then broadened.

(3) The third child population: the search space U^3 is the promising space reduced by the roulette wheel selection model to $U_i^{pr} = [\underline{x}_i^{pr}, \overline{x}_i^{pr}]$, and then broadened. For example, each variable is partitioned into five segments. According to the probability model of a roulette wheel, from each segment, the individuals that have better average function values are assigned higher probabilities of being selected.

In this paper, in order to simulate the decision-making behavior of human society, four parameters are set: the evaluated generation number, the number of better individuals, the broaden ratio, and the number of partitioned segments. We set the number of partitioned segments as 5. To assess the search capability of this method, we show examples of the three major parameters in the next sections and demonstrate the advantages of the grouping search strategy. The flowchart of IGCMEA is shown in Figure 3:

(1) Evaluated generation number, N_E : After every N_E generations, the grouping strategy is applied to the population to simulate the regular group meetings.

(2) Number of better individuals, N_B : In the evaluated population, the individuals with better function values are selected. Their variables, lower and upper bounds are used to confine the promising space, simulating the choices in a meeting.

(3) Broaden ratio B_r : Once the promising space is determined, the promising space is broadened, simulating flexible decisions, in order to improve the population's optimization performance.

4. Examples and discussion

In this section we use IGCMEA to optimize sevenbenchmarkfunctions and bridge structures.

4.1 IGCMEA for the 100-dimensionalsevenbenchmarkfunctions

In order to understand the ability of the optimumalgorithm proposed in this paper for the global optimization search, seven benchmarking functions, which are often cited in the literature, are selected for test examples.The seven benchmark functions are shown in Table 1 and the 7th3D graphical diagrams of two-dimensional are shown in Figure 4 respectively

The settings of IGCMEA for solving this function are:

(1) The mutation strategy settings:

- In each generation, randomly select from the following three strategies

(1) DE/best/1

(2) DE/rand-to-best/1

(3) DE/rand/2

(2) The parameters are as follows:

- dimension n: 100
- number of populations NP: 800 (NP = 5 ~ 10 × n [14])
- maximum number of generations MaxGen: 6000
- mutation factor F = 0.5 ~ 0.6
- crossover rate CR = 0.3 [14]
- evaluated generation numbers are 100,200,300
- broaden ratios of promising space are 10,20,30

(3) The number of better individuals is set to 400

The results of comparison DE, GCMEA and IGCMEAare shown in Tables 2.From Table 2 it can be seen that the optimization tool of IGCMEA in 100-dimension $F_1(X)$~$F_6(X)$ problems is still very stable compared to the DE and GCMEA. The GCMEA compared to DE in the $F_3(X)$

problem, the solving performance is still no improvement.On 100-dimensional $F_7(X)$ optimization problem, IGCMEA average solution -40498.72 is better than DE and GCMEA, and it was observed from the standard deviation that IGCMEA with better cohesion could yield better results.

Taking $F_7(X)$ as an example, through the average distance between the ethnic groupand best solution of each generation ethnic group and the average distance between the ethnic groupand analytical solution, we try to understand the dynamic behavior of ethnic group movement.Figure 5 are the convergence diagram of DE, GCMEA and IGCMEA. Figure6 and7 are those average distance of the ethnic groupand best solution of each generation ethnic group $d_{pg}(\%)$ and the average distance betweenethnic group and analytical solution. $d_{pa}(\%)$ for the Schwefel function $F_7(X)$ respectively. The definition of two average distances $d_{pg}(\%)$ and $d_{pa}(\%)$ are

$$d_{pg}\,(\%)=\frac{\overline{d_{pg}}}{L}\times 100\% \quad (13)$$

$$d_{pa}\,(\%)=\frac{\overline{d_{pa}}}{L}\times 100\% \quad (14)$$

$\overline{d_{pg}}$ is the average distance between individual of each generation ethnic groups and the best individualsX^{gbest} , N_p number of group.

$$\bar{d}_{pg}=\frac{\sum_{i=1}^{NP}\left\|X_i-X^{gbest}\right\|}{N_p} \quad (15)$$

$\overline{d_{pa}}$ is the average distance betweenethnic group and analytical solution.

$$\bar{d}_{pa}=\frac{\sum_{i=1}^{NP}\left\|X_i-X^*\right\|}{N_p} \quad (16)$$

$L=\sqrt{\sum_{i=1}^{n}\left(\overline{x_i}-\underline{x_i}\right)^2}$, $\overline{x_i}$ and $\underline{x_i}$ are upper bounds and lower bounds of design variable x_i .

From the results, IGCMEA combines GCMEA has the integration and the best orientation, in the early stage of the search that is able to detect the important extreme value, with the best potential to reach the global optimization. The IGCMEA exhibit robust convergence trend, and gradually move to better ethnic space.The average distance of IGCMEA is about 27.16%, while DE and GCMEA can only search for 39.02% and 42.24% respectively.

4.2 Shape optimization for plane frame bridges

This structure problem is shown in Figure 8 [15]. For a simply supported plane frame bridge, the objective function of this problem is to find the top coordinates of five supporting columns, i.e.

designing the shape of the bridge, so that the maximum moment M_{max} of the bridge structure is at the minimum. The structure has 100kN point loading applied at nodes 2, 4, 6, 8, 10. The sectional area of each part is shown in Table 3. Young's modulus is 210GPa. No other restrictions on size or response are enforced in this problem. There are five design variables in total, namely the distances from nodes 3, 5, 7, 9 and 11 to the lower chord. Therefore, the mathematical optimization model is as follows:

$$\textbf{\textit{Minimize}} \quad M_{max} \tag{17}$$

The settings of IGCMEA for solving this optimization are:

(1) The mutation strategy settings:

- In each generation, randomly select from the following three strategies

(1) DE/best/1

(2) DE/rand-to-best/1

(3) DE/rand/2

(2) The parameters are as follows:

- dimension *n*: 5
- number of groups NP: 300
- maximum number of generations MaxGen: 100
- mutation factor F = 0.5 ~ 0.6
- crossover rate CR = 0.3
- search space: the lower and upper bounds of all variables are 1m to 2.5m
- evaluated generation number: 5
- number of better individuals: 150
- broaden ratio: 10

The results are shown in Figures 9 and 10, and Table 4.

From Table 4 we can see that the result of IGCMEA in the COMSOL [16,17] environment is 3674.6 (Nm), which better than the result of steepest descent method as used by Wang [15].

Moreover, we re-analyzed the optimization result of D. Wang in the COMSOL software, and obtained the maximum moment value of 3767.1 (Nm).

5. Conclusions

(1) In this paper, we successfully implemented an automatic optimization interface program between the finite element analysis software COMSOL Multiphysics and MATLAB. Through the intuitive and simple graphical user interface of COMSOL Multiphysics and the convenient editing environment of MATLAB, we are provided with a development platform for optimum structure design.

(2) In the test of the benchmark Function, the proposed Intelligent Garbage Can Model Evolution Algorithm (IGCMEA) showed its stability in searching for the global optimum. This novel algorithm also achieved satisfactory results when applied to the shape optimization problem of plane frame

bridges.

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Table 1. The n-dimensional benchmark Functions and the exact solutions.

$F_1(X)$	Spherical	$X^* = 0, \mathrm{F}(X^*) = 0$ $\mathrm{F}_1(\mathrm{X}) = \sum_{i=1}^{n} x_i^2$
$F_2(X)$	Quadric	$X^* = 0, \mathrm{F}(X^*) = 0$ $\mathrm{F}_2(\mathrm{X}) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2$
$F_3(X)$	Rosenbrock	$X^* = 1, F(X^*) = 0$ $\mathrm{F}_3(\mathrm{X}) = \sum_{i=1}^{n} (100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2)$
$F_4(X)$	Ackley	$X^* = 0, \mathrm{F}(X^*) = 0$ $\mathrm{F}_4(\mathrm{X}) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2} \right) - \exp\left(\frac{1}{n} \sum_{i=1}^{n} \cos(2\pi x_i) \right) + 2$
$F_5(X)$	Griewank	$X^* = 0, \mathrm{F}(X^*) = 0$ $\mathrm{F}_5(\mathrm{X}) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}} \right) + 1$
$F_6(X)$	Rastrigin	$X^* = 0, \mathrm{F}(X^*) = 0$ $\mathrm{F}_6(\mathrm{X}) = \sum_{i=1}^{n} \left(x_i^2 - 10 \cos(2\pi x_i) + 10 \right)$
$F_7(X)$	Schwefel	$X^* = (-420.9687, -420.9687, \cdots, -420.9687), \mathrm{F}(X^*) = -12569.5$ $\mathrm{F}_7(\mathrm{X}) = \sum_{i=1}^{n} x_i \sin\left(\sqrt{\lvert x_i \rvert} \right)$

Table 2. Comparisions of DE, GCMEA and IGCMEA applied in the 100-dimensional Rosenbrock Functions.

Function / method		$F_1(x)$	$F_2(x)$	$F_3(x)$	$F_4(x)$	$F_5(x)$	$F_6(x)$	$F_7(x)$
DE	f_{av}	9.65E-06	5.75E-04	3.99E-01	9.81E-06	7.48E-04	9.51E-06	-38534.63
	f_b	9.13E-06	2.51E-04	8.11E-06	9.56E-06	8.89E-06	8.47E-06	-39490.03
	f_w	9.99E-06	1.36E-03	3.99	9.98E-06	7.39E-03	9.95E-06	-37535.81
	σ_f	2.79E-07	3.19E-04	1.19	1.52E-07	2.22E-03	3.92E-07	540.04
GCMEA	f_{av}	4.83E-06	4.86E-06	4.22E-01	4.79E-06	4.89E-06	4.81E-06	-39092.74
	f_b	9.52E-06	9.42E-06	8.99E-06	9.05E-06	9.46E-06	9.19E-06	-39781.19
	f_w	9.84E-06	9.94E-06	4.01	9.90E-06	9.94E-06	9.98E-06	-38360.67
	σ_f	3.42E-06	3.44E-06	1.24	3.40E-06	3.46E-06	3.41E-06	568.90
IGCMEA	f_{av}	4.82E-06	4.92E-06	4.81E-06	4.88E-06	4.56E-06	4.83E-06	-40498.72
	f_b	9.48E-06	9.75E-06	9.18E-06	9.60E-06	8.75E-06	9.41E-06	-40948.76
	f_w	9.88E-06	9.94E-06	9.86E-06	9.95E-06	9.44E-06	9.98E-06	-39763.41
	σ_f	3.41E-06	3.48E-06	3.41E-06	3.45E-06	3.23E-06	3.42E-06	436.15
exact solution		0	0	0	0	0	0	-41898.30

Where: f_{av} is average soluion, f_b is best solution, f_w is worst solution, σ_f is standard deviation of solutions.

Table 3. Sectional areas of the bridge elements of plane frame bridges

Element groups	Upper chord	Lower chord	Columns
Shape	Tubular	Rectangle	Cylinder
Size	Outer radius: 18cm Inner radius: 17cm	Width: 10cm Height: 15cm	Diameter: 5cm

Table 4. Comparison results of IGCMEA and the literature [15] for the plane frame bridge problem

Design variables	D. Wang[15]	IGCMEA
#1 element top coordinate (Joint 2 to 3)(m)	1.3272	1.3442
#2 element top coordinate (Joint 4 to 5)(m)	2.1252	2.1528
#3 element top coordinate (Joint 6 to 7)(m)	2.3934	2.4239
#4 element top coordinate (Joint 8 to 9)(m)	2.1252	2.1528
#5 element top coordinate (Joint 10 to 11)(m)	1.3272	1.3442
F(X)(N-m)	4350	3674.6

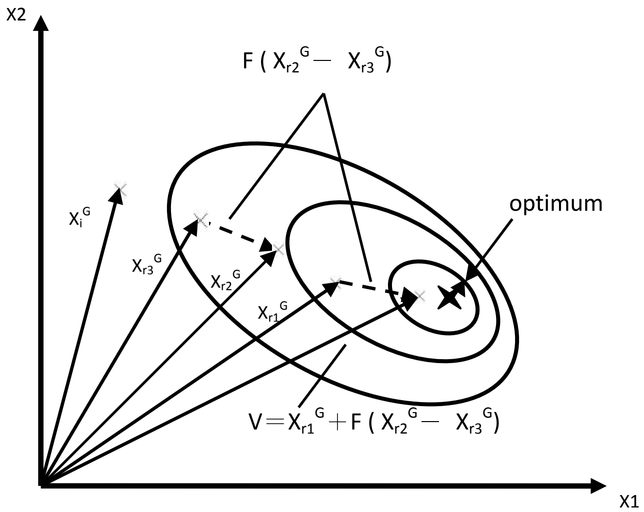


Figure 1. Generation of a disturbance vector by mutations[10]

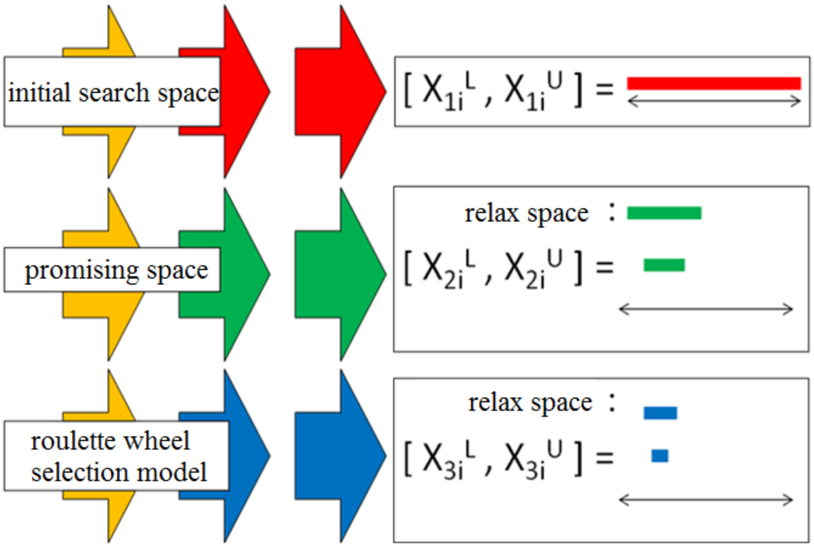


Figure 2. Groups of IGCMEA grouping search strategy

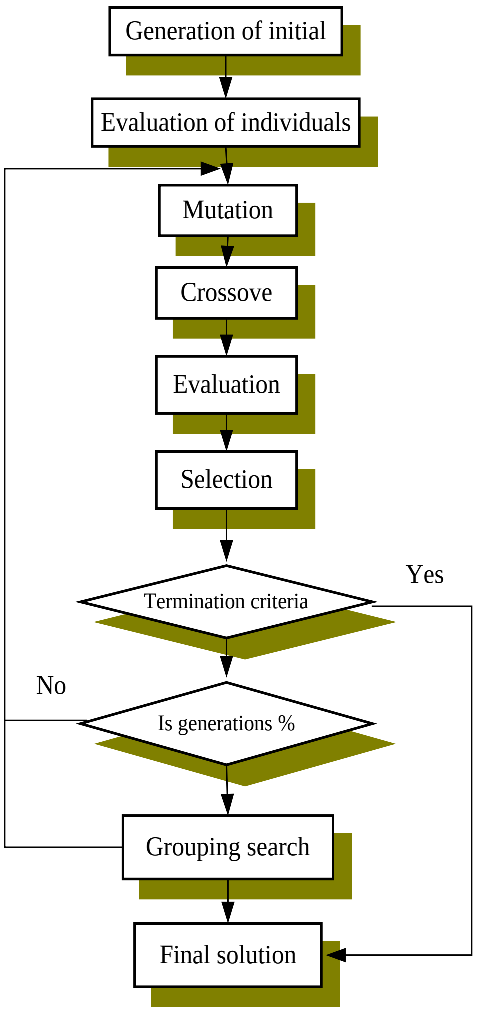
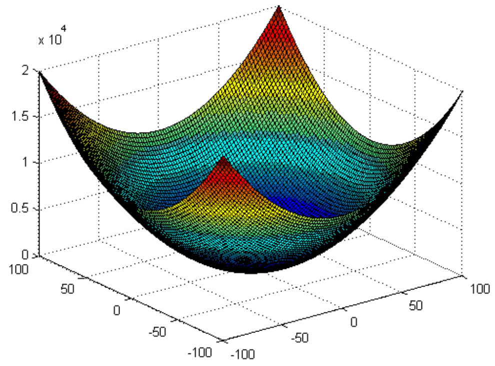
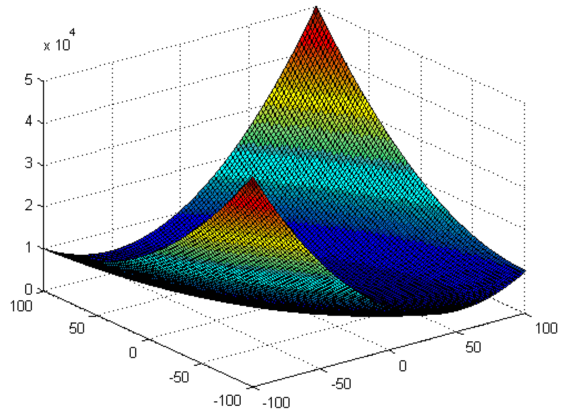
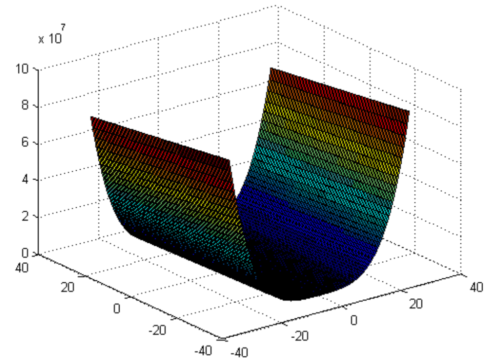
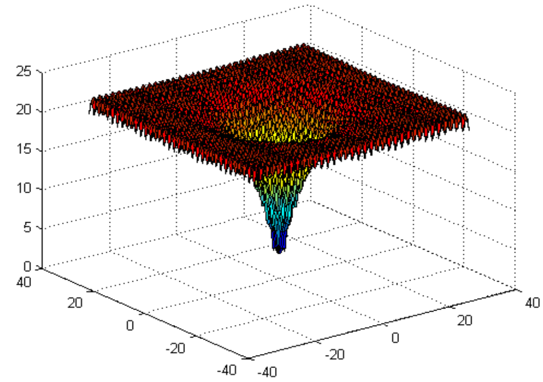


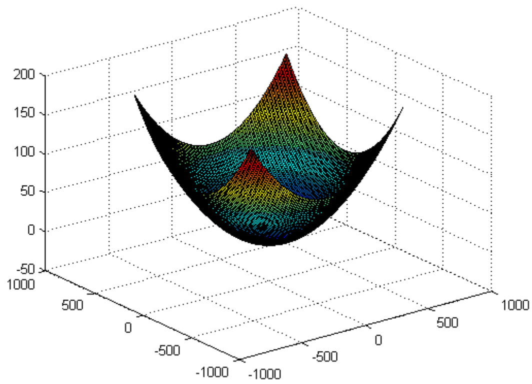
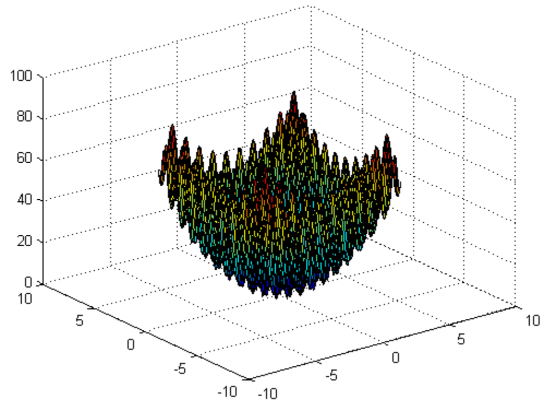
Figure 3. Flow chart of IGCMEA

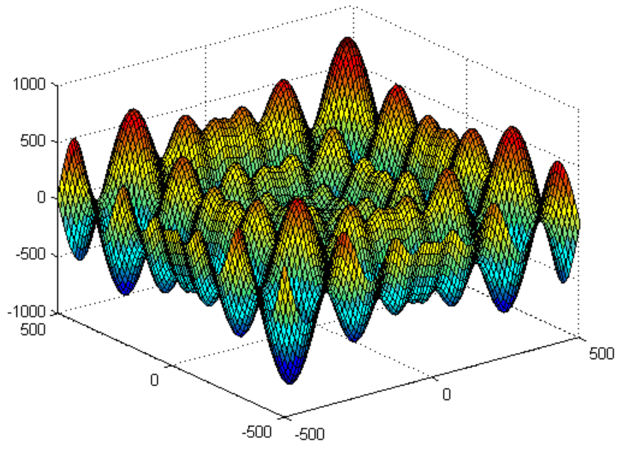
(a) $F_1(X)$ Spherical(b) $F_2(X)$ Quadric

(c) $F_3(X)$ Rosenbrock (d) $F_4(X)$ Ackley

(e) $F_5(X)$ Griewank(f) $F_6(X)$ Rastrigin



(g) $F_7(X)$ Schwefel

Figure 4. 3D graphical diagram of two-dimensional $F_1(X) \sim F_7(X)$

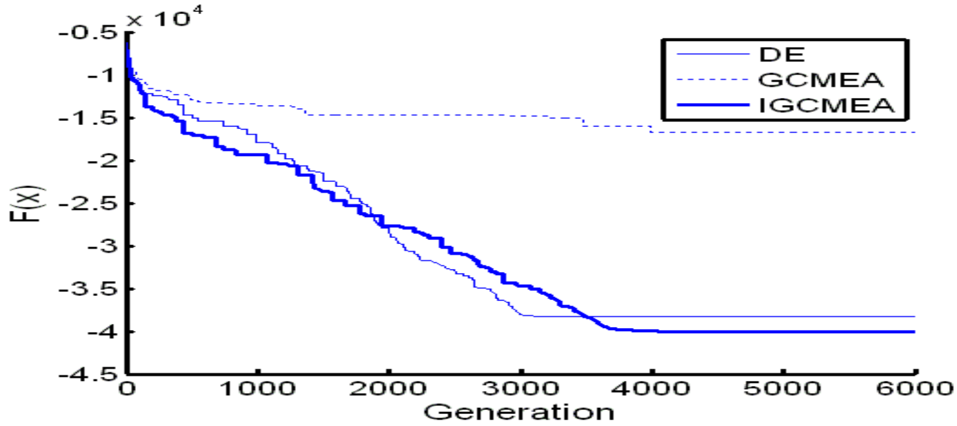


Figure 5. Search convergence diagram of 100-dimensional Schwefel function $F_7(x)$.

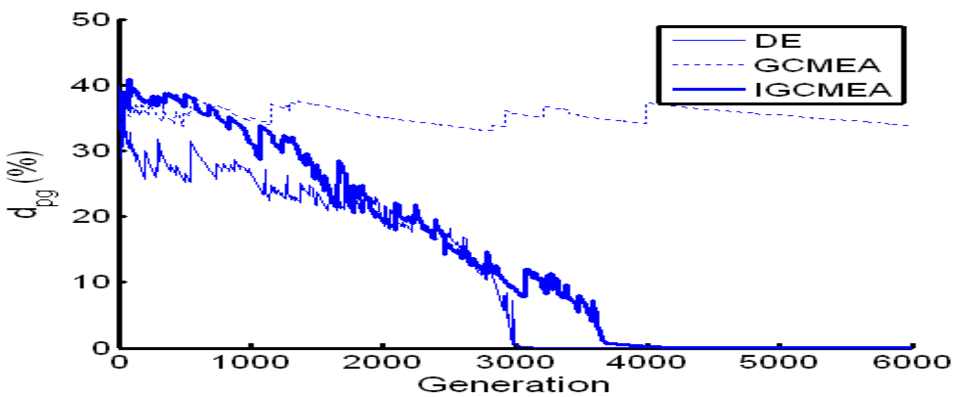


Figure 6. The average distance d_{pg}between the ethnic group and best solution of each generation for $F_7(X)$.

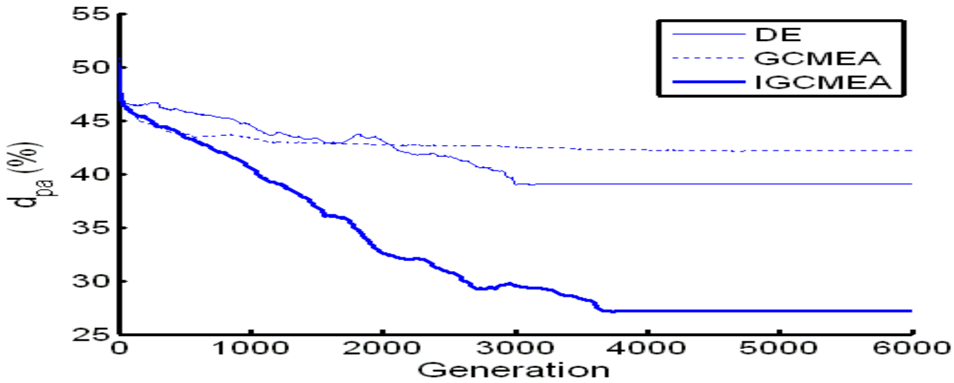


Figure 7. The average distance d_{pa}between the ethnic group and best analytical solution for $F_7(X)$.

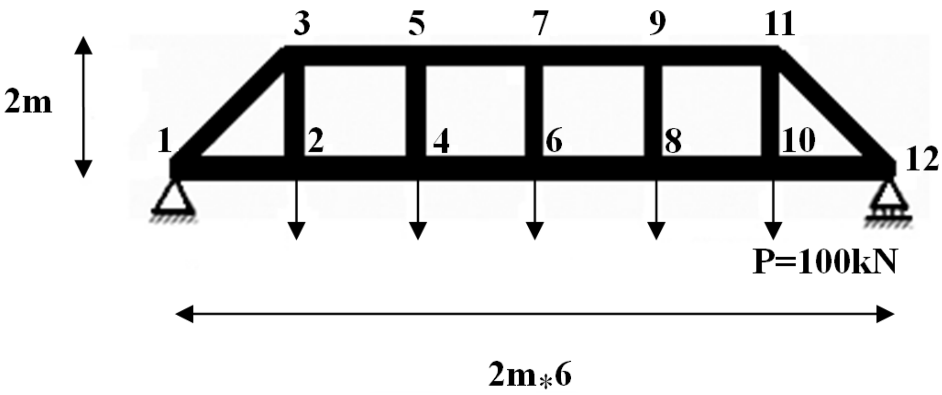


Figure 8. Structure of plane frame bridge

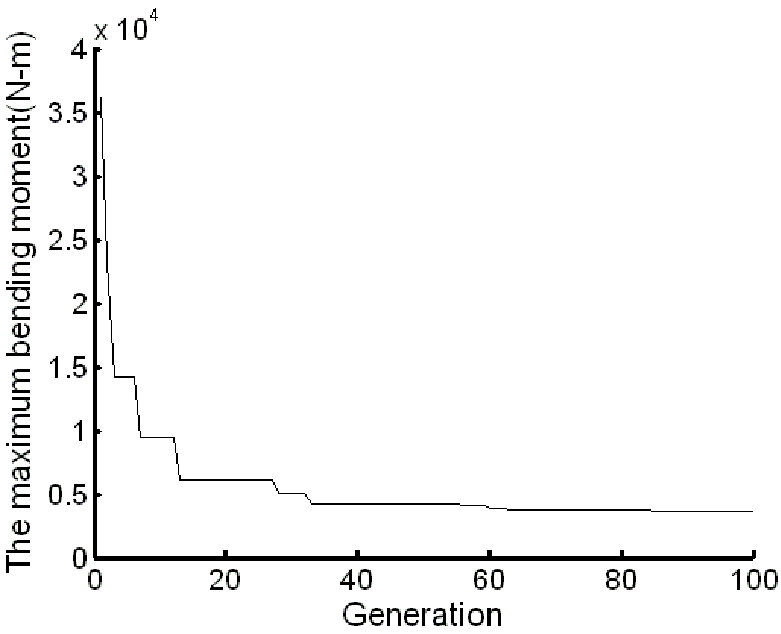


Figure 9. Convergence of the shape optimization of plane frame bridges

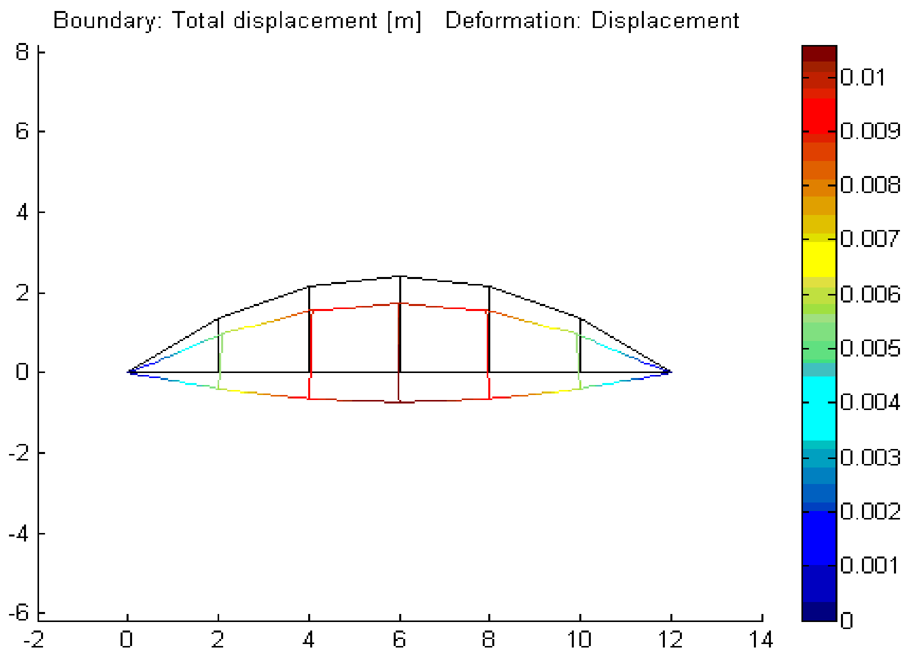


Figure 10. The deformation of the optimum design of plane frame bridges